

Planning in forest ecosystems: the role of models

Oscar García



forestgrowth.unbc.ca

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Title is somewhat misleading. I changed my mind, instead of speaking generalities, decided to try an experiment: See if in 15 minutes I could explain a modelling approach not all that obvious or well-known to many people. Hope it to be obvious toward the end.

Outline

- Introduction
- Dynamical systems and growth models
- An example
- Further work
- Conclusions



Why models?

- Long-term planning. Trial-and-error not effective. Mathematical models
- Forest regulation. Yield tables



Because of long time horizons, experience and experimentation not as useful in forest planning as in other instances. Models needed for prediction.

National/regional/forest-level planning addressed by classical regulation theory, and newer decision support systems. A pre-requisite is growth forecasting, by yield tables and their modern successors. Although the principles apply also to forest-level models, we will focus here on stand-level examples.

Growth and yield

- New challenges:
 - Varying environment
 - More than timber
 - More than final yield
 - Boreal: scarce data (Canada)
 - Thinning? Disturbances
 - Adaptive management
- Dynamical systems theory to the rescue!

Age	I	II	III	IV	V	VI	VII	VIII	IX	X
1	100	100	100	100	100	100	100	100	100	100
2	100	100	100	100	100	100	100	100	100	100
3	100	100	100	100	100	100	100	100	100	100
4	100	100	100	100	100	100	100	100	100	100
5	100	100	100	100	100	100	100	100	100	100
6	100	100	100	100	100	100	100	100	100	100
7	100	100	100	100	100	100	100	100	100	100
8	100	100	100	100	100	100	100	100	100	100
9	100	100	100	100	100	100	100	100	100	100
10	100	100	100	100	100	100	100	100	100	100
11	100	100	100	100	100	100	100	100	100	100
12	100	100	100	100	100	100	100	100	100	100
13	100	100	100	100	100	100	100	100	100	100
14	100	100	100	100	100	100	100	100	100	100
15	100	100	100	100	100	100	100	100	100	100
16	100	100	100	100	100	100	100	100	100	100
17	100	100	100	100	100	100	100	100	100	100
18	100	100	100	100	100	100	100	100	100	100
19	100	100	100	100	100	100	100	100	100	100
20	100	100	100	100	100	100	100	100	100	100
21	100	100	100	100	100	100	100	100	100	100
22	100	100	100	100	100	100	100	100	100	100
23	100	100	100	100	100	100	100	100	100	100
24	100	100	100	100	100	100	100	100	100	100
25	100	100	100	100	100	100	100	100	100	100

This yield table, published by Cotta in 1821, shows volume as a function of age, for various site qualities.

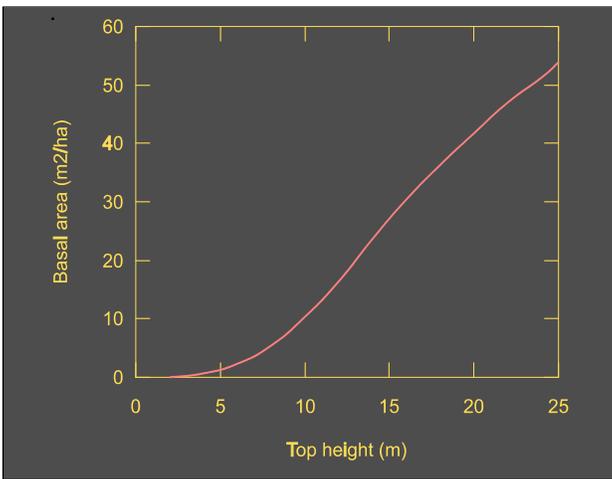
Old yield tables or traditional growth and yield models are often all that is needed. But fail to address satisfactorily some issues that are becoming important:

- Response to changes in climate or other factors
- Carbon sequestering and other ecosystem services
- For carbon, etc., interest in all ages, not just the final timber harvest
- Insufficient long-term data, for instance in Canadian boreal forests
- Interest in response to thinnings or other management or natural disturbances

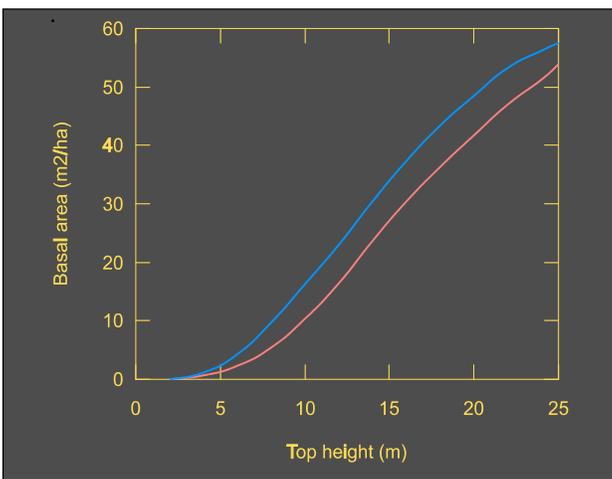
disturbances

- Updates from inventory information

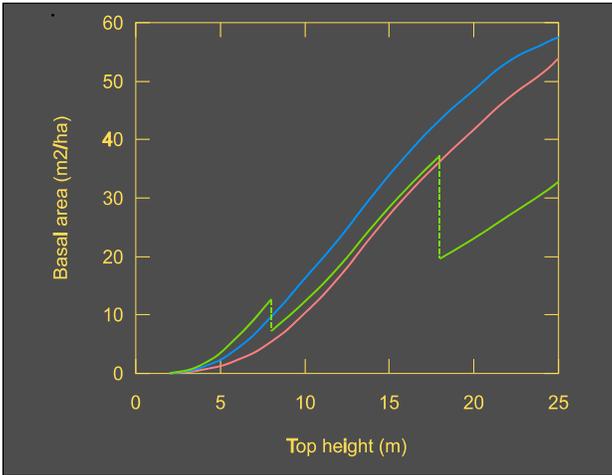
Yield tables are essentially fixed functions of time, typically including volume, height, basal area, number of trees, for various ages. With a revised point of view it is possible to handle all the above.



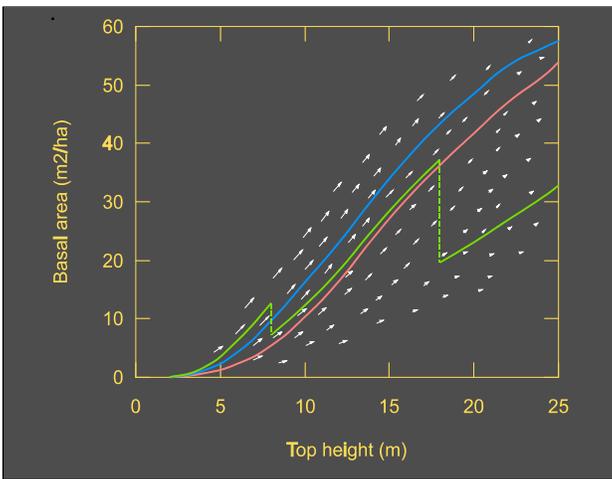
Consider the height and basal area columns from a yield table. Ignore differences in site quality for now. The trajectory followed by a stand can be represented as in this graph, with points on the curve corresponding to the various ages.



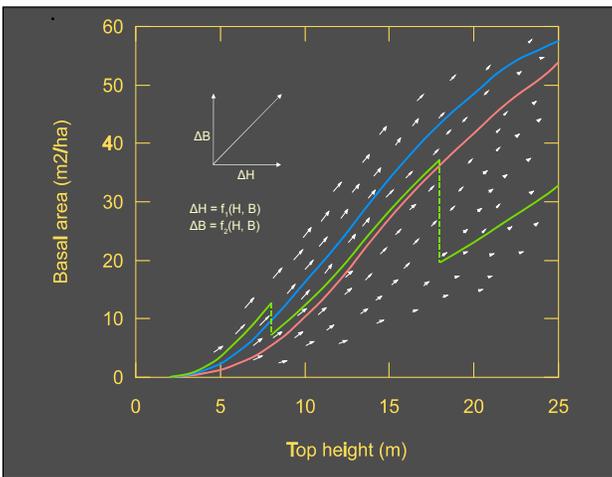
Different initial densities can be handled easily, generating different curves.



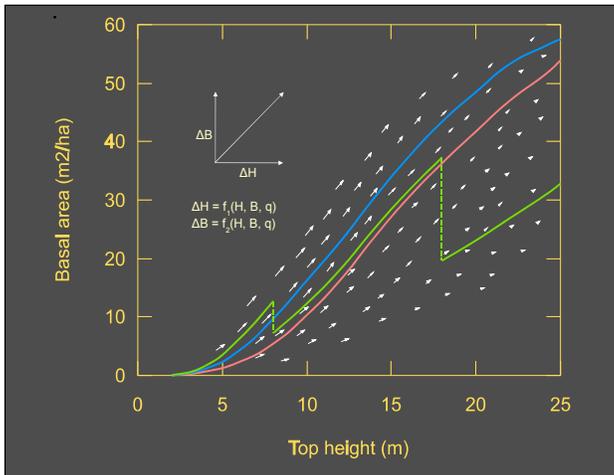
But what about thinnings, as in the green trajectory?
 Frequent light thinnings, as in European practice, are often smoothed-out and approximated by curves from managed yield tables. That is not good enough with one or two heavy thinnings as shown here, typical of plantation forestry in the Southern Hemisphere or in the U.S. South.
 Updating projected yields with inventory data produces similar discontinuities.



A simple trick solves the problem. Instead of trying to model the trajectories directly, we predict the change for a small time interval at every point. The white arrows indicate the change of state of the stand (height and basal area), depending on the current state.
 Trajectories are constructed by following the arrows. After a disturbance, we follow the arrows corresponding to the new point. Any combination of thinning times and intensities (or updates, or other disturbances) can be simulated in this way.

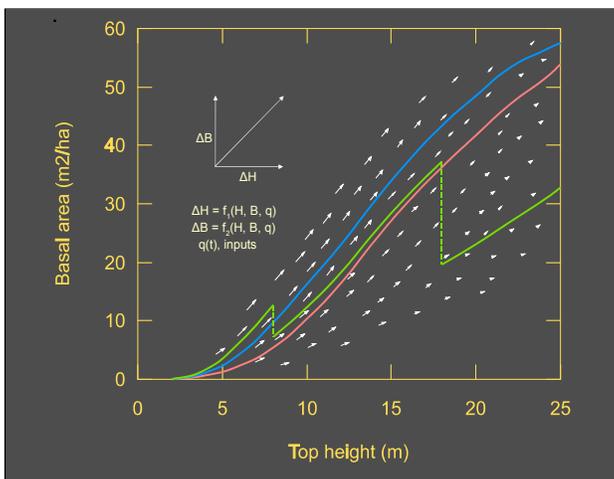


The model predicts the periodic (or annual, or infinitesimal) change in each of the two state variables, height and basal area, as a function of the current state.
 In the absence of disturbances, the trajectory is computed by iterating these relationships. A disturbance causes an instantaneous change of state, after which the same procedure is applied.



The rate of change equations may depend on other variables (“inputs”), such as a site quality q .

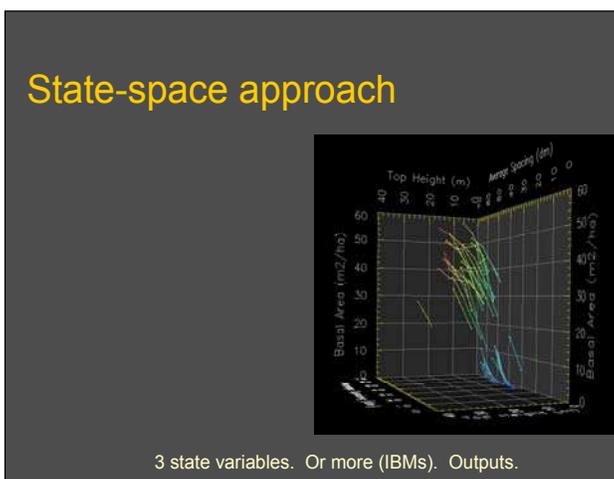
Note that q does not need to be a constant, it can be time-dependent. All that happens is that the length and/or direction of the arrows vary over time. We simply use whatever arrow happens to be at the current point when we get there. Thus, one can model changes in climate or nutrients.



One may be interested in things other than the state variables H and B , such as volume. These “outputs” may be computed from the current state. For instance, volume per hectare is commonly estimated from a regression on H and B , a stand volume table or function.

All the above depends on the assumption that, to an appropriate degree of approximation, the behaviour of the state variables is determined by their current values. However, two stands with the same H and B , but different number of trees per hectare, may have different basal area growth. Also, merchantable volumes are affected by average tree size in addition to H and B . The model can be improved by adding the number of trees per hectare (or average spacing, or mean diameter) as a third state variable.

The principles are the same, but now there is a 3-dimensional state space (and 3 rate equations).



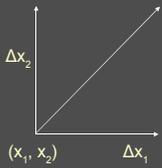
These are observed trajectories of permanent sample plots, with average spacing as a 3rd state variable. On the left it is radiata pine in New Zealand; the purple lines correspond to thinnings. On the right, unthinned interior spruce in British Columbia.

A stand immediately after thinning may not fully occupy the site, growing slightly less than another with the same values of the three variables but not recently thinned. Therefore, more accuracy might require a fourth variable, especially with heavy thinning and pruning.

Conceptually, individual-based models can be described in the same way, but they may contain hundreds of state variables, at least one diameter for each tree in a sample.

It is worth noting that the fact that rates and outputs are determined by the current state, more than an assumption is a definition. In principle, it is always possible to add variables until they constitute a proper state vector, to any desired degree of approximation. The appropriate dimensionality is a practical compromise between accuracy, parsimony, available data, and other considerations.

Dynamical systems



$$\Delta x_1 = f_1(x_1, x_2, u)$$

$$\Delta x_2 = f_2(x_1, x_2, u)$$

$$y = g(x_1, x_2)$$

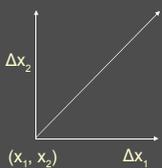
- x** : state variables
- u** : input(s)
- y** : output(s)

None of this is new. The basic idea of modelling through rates of change probably originated with Isaac Newton in the 17th Century, and it is taken for granted in physics and engineering. In the 1960's, System Theory abstracted the general principles from the physical details. There were also contributions from Cybernetics and Optimal Control Theory. Nowadays the subject is part of Dynamical Systems Theory, although the current obsession with chaotic behaviour is not relevant to us, only the basic elements (look up "dynamical system" in Wikipedia).

These days the approach is commonly used in many fields, but not in forestry. Perhaps because of a long tradition of thinking in terms of functions of time.

The general idea is to describe a system by a number of state variables. The rate of change of these is given by a set of difference equations, in discrete time, or differential equations in continuous time. These depend on the current state, and possibly also on one or more input variables. Outputs are functions of the current state.

Dynamical systems



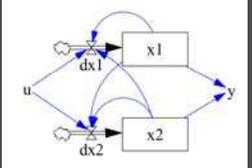
$$\Delta x_1 = f_1(x_1, x_2, u)$$

$$\Delta x_2 = f_2(x_1, x_2, u)$$

$$y = g(x_1, x_2)$$

- x** : state variables
- u** : input(s)
- y** : output(s)

System Dynamics, Forrester, stocks/flows, levels/rates



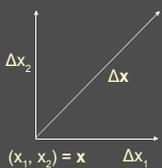
input rate state output

The System Dynamics graphical notation of Forrester is useful for communication, especially with mathematically averse/impaired researchers and students. There is also software that performs simulations based on the diagrams, requiring little or no mathematical or programming knowledge.

It is usually explained in terms of blocks or compartments that contain a stock of some "stuff". The stock changes at a rate given by a flow of stuff in or out of the box, represented by a pipe with a control valve. The dependence of the flow on various stocks and auxiliary variables (parameters and/or inputs) is indicated by arrows. Arrows also define outputs as functions of stocks.

The stock/flow analogy might be stretched a bit to far when using variables such as height. A more general level/rate terminology may then be more appropriate. Essentially, the stocks or levels correspond to state variables, and the flows or rates to difference or differential equations.

Dynamical systems



$$\Delta x_1 = f_1(x_1, x_2, u)$$

$$\Delta x_2 = f_2(x_1, x_2, u)$$

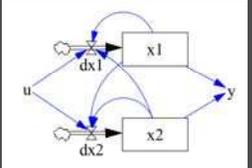
$$y = g(x_1, x_2)$$

- x** : state
- u** : input
- y** : output

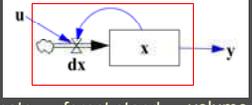
$$\Delta x = f(x, u)$$

$$y = g(x)$$

System Dynamics, Forrester, stocks/flows, levels/rates



input rate state output

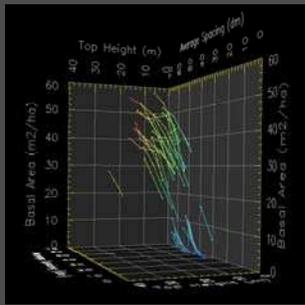


climate, forest stand, volume, etc. (thinnings) carbon

A lot of writing can be saved by using a single symbol as shorthand for a list of numbers, a "vector", usually distinguished by bold face or underlining.

By the way, the difference equations should not be confused with the terminology of "algebraic difference equations" used in forest modelling. Although related, mathematically these are neither difference equations nor algebraic.

Growth models -- Data



Empirical. Process-based. Theory-based ("hybrid").

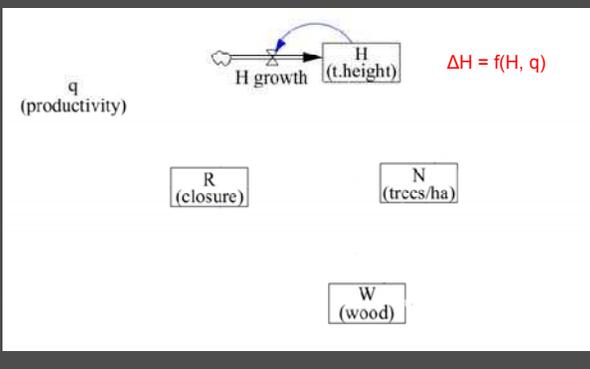
How does this work in growth modelling?

With adequate data, as in the New Zealand example (left), flexible purely empirical equations with three or four state variables have been used. The aim was to describe the observed behaviour free of the influence of preconceived ideas.

Sparse data (right) requires guidance from eco-physiological theory and experience from elsewhere. Process-based modelling tries to synthesize that knowledge. Such models are important research tools but, at least at present, tend to be too detailed and unreliable to be used directly in operational management. The knowledge can be used, however, to guide the development of semi-empirical models consistent with the biological principles. These theory-inspired models are often called "hybrid", although all real models are hybrid to some degree, and the word tends to mean different things to different people.

We describe briefly a parsimonious and biologically consistent growth model for the British Columbia spruce data shown on the right. The same model structure was tested successfully on an extensive data set for loblolly pine in the southern USA.

A growth model



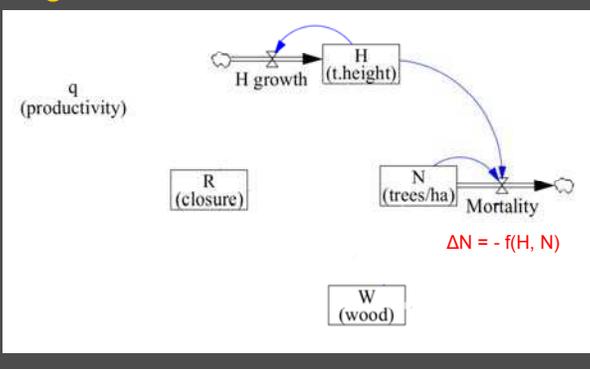
Four state variables: top height, number of trees per hectare, $W = B * H$ that is roughly linearly related to stem volume or biomass, and a variable R representing relative canopy closure.

Height growth rate depends on current height and site quality. A self-contained sub-system, corresponding to a conventional site index model.

In some models growth is assumed to depend on age. Although both variables are highly correlated, physiology suggests that the dominant factor is size, not age.

The arrows indicating the influence of the site parameter q on all the rates will be omitted.

A growth model

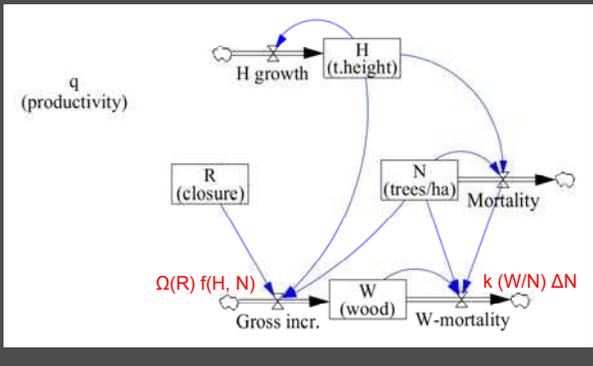


N decreases at a rate depending on H and N (and q , which will be omitted to simplify).

As done before, age is excluded as a causal variable.

Also excluded are mean diameter, basal area, or stem volume, variables commonly used in self-thinning relationships. These variables represent the amount of wood accumulated on the stem, much of it dead as heartwood. There are no good biological reasons why it should affect growth or mortality.

A growth model



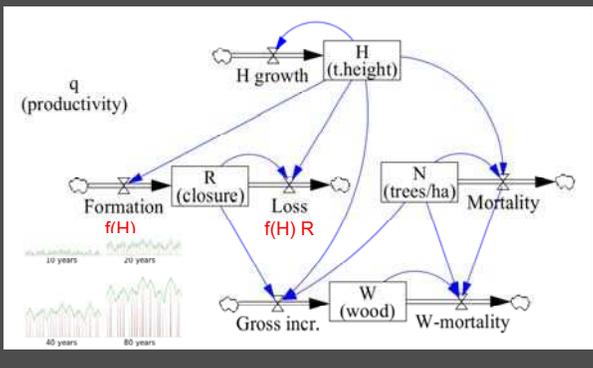
The change in W equals gross increment minus mortality.

The mortality is the mortality in number of trees times the mean tree size, reduced by a factor representing the size of dying trees relative to those that live.

In a fully closed stand ($R=1$), gross increment may depend on H and N . Again, we exclude age and W for biological reasons.

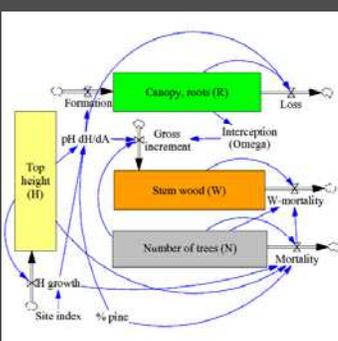
If the stand is not fully closed, because it is young or recently thinned, growth is reduced by an “occupancy” factor that depends on closure. One may think of closure as (relative) amount of foliage and fine roots, and occupancy as interception of resources (light, water, nutrients).

A growth model



Finally, a model for the changes in closure is needed. Thinking of the foliage, it initially increases as height increases. As it approaches closure, it is lost from the crown base and older leaves/needles, tending to reach a balance.

A growth model



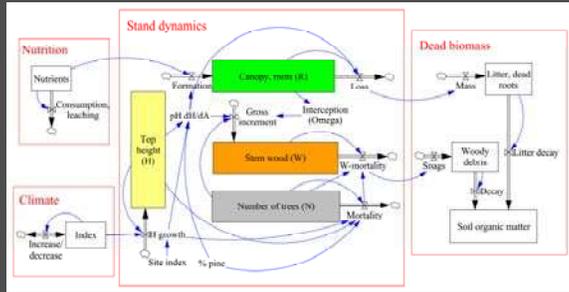
The complete model, re-drawn in a prettier arrangement.

On the right, the equations, with the coefficients for the pine version.

These are differential equations (continuous time), rather than difference equations (discrete). Difference equations seem easier at first sight, but things get messy when the measurement intervals do not match the projection intervals (and both intervals may vary).

By an extension of Eichhorn's Law, rates relative to height increment are independent of site. This proved to be a good approximation.

Environment, carbon -- Modules



García, Burkhardt & Amateis (2011)

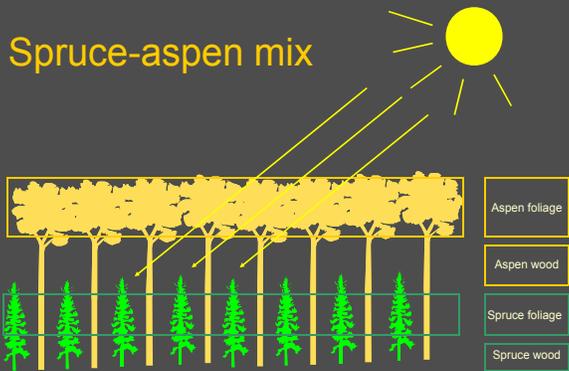
Spruce, aspen → Two-storied mix

The model could be linked to modules describing environmental changes and secondary processes such as carbon cycling. The modules shown here are simplified placeholders for illustration only.

A modular approach should be better than monolithic models with detailed environmental components but simplistic stand dynamics, or vice-versa.

More in G, B & A (2011) *Forest Ecology and Management* 262(11), 2035–2041, and <http://forestgrowth.unbc.ca/lobdyn>

Spruce-aspen mix



A similar model for aspen is near completion. The next step is to put the spruce and aspen models together to describe mixed-species stands. Initially perhaps two-storied stands. Coupled through the occupancy, e.g., light interception.

Conclusions

- Long-term planning requires mathematical modelling
- State-space approach allows for disturbances and a varying environment
- Biologically-consistent semi-empirical models can be parsimonious and robust
- Modular strategies for improving balance

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Consistency includes balancing gain and losses, and the exclusion of age, diameter, or basal area as drivers.

Everybody's research area is the most important, and tends to dominate model development. Separate modules can be worked on by the appropriate experts, contributing to more balanced models.